# AN EXPERIMENTAL STUDY OF VARIABILITY IN LEARNING

BY

SOLOMON E. ASCH

Tutor, Department of Philosophy, Brooklyn College

ARCHIVES OF PSYCHOLOGY

R. S. WOODWORTH, EDITOR

*No. 143*



NEW YORK

October, 1932

34. Quickness of Learning and Retentiveness: D. O. LYON. 50c. (Cl., 75c.)
35. Overcoming of Distraction: J. J. B. MORGAN. 75c. (Cl., $1.00.)

ing: H. E. JONES. $1.25.

69. Influence of Treatment for Intestinal Toxemia on Mental and Motor Efficiency: A. E. PAULSEN. $1.00.

(*Continued on inside back cover.*)

# AN EXPERIMENTAL STUDY OF VARIABILITY IN LEARNING



BY
SOLOMON E. ~~ASCH~~
Tutor, Department of Philosophy, Brooklyn College

ARCHIVES OF PSYCHOLOGY
R. S. WOODWORTH, EDITOR
*No. 143*

NEW YORK
October, 1932

# TABLE OF CONTENTS

# An Experimental Study of Variability in Learning

## CHAPTER I

## THE PROBLEM*

*I. Introduction*

That each individual fluctuates in ability from moment to moment is as obvious an observation as that individuals differ from each other. His tendency to vary from his own level of performance is as characteristic of each person as is the level from which he varies. A person becomes successively stronger and weaker, more and less intelligent, better and worse. In short, his powers wax and wane as the complex influences playing upon him from within and without change in number, intensity, and direction.

Do these variations occur without rhyme or reason, or can any invariant relations be discerned among them? Because of its very pervasiveness, this characteristic of mental activity has implications bearing directly upon issues that have been raised in systematic psychology. Psychologists have accordingly turned their attention to this aspect of behavior in the hope of relating it to certain other traits within the individual. There have been attempts to connect the amount of variability an individual manifests to the amount of integration he possesses, or to general emotional stability. The problem, whether the variability of an individual is specific to each task, or whether there exists an independent trait of variability, indicates its bearing upon a systematic analysis of the organization of mental abilities. The amount of variation in a function, and its mode of distribution, have been recognized as important questions for the theory of mental measurement. The relation of the amount of variability in a mental function to the degree of ability in that function is related to certain theories of the constitution and operation of intellectual abilities. And the conditions of its occurrence, the factors that increase or decrease its magnitude are important both for a precise account of the relationships obtaining in any experimental situation as well as for a clearer interpretation of differences between individuals.

* This study is one of a series under the general direction of Professor H. E. Garrett. The author wishes to take this opportunity to express his gratitude to Professor Garrett for the suggestion of the problem and for constant helpfulness at all stages of the work.

*II. Purpose of the Present Investigation*

The present study represents an attack upon a number of specific problems in the variation of individuals from their own norm of performance. The object of this experiment is to determine whether there is evidence for any relation between variations in a number of different mental tasks, within an individual. If an individual is tested repeatedly with a number of mental tasks, when these are performed within a half-hour period, will each deviation be chance or will it be related to deviations in the other tasks? Since each deviation of an individual up or down from his norm represents a change in efficiency, our problem may read: Are variations in efficiency determined by the operation of systematic factors which persist and affect future performances within the same half-hour, or are they the result of chance stimulation, affecting only the scores from which they were measured?

Each of the tests selected was repeated a number of times. There was considerable improvement in each of the performances. The variability here measured is consequently variability during learning. This choice of tests of learning was decided upon because variability in learning functions has not been adequately measured, and because little is known concerning its magnitude, distribution, and relationships.

In view of the nature of the tasks chosen, our problem may be restated more specifically, as follows:

1. Is there any relation between variations in a number of different learning processes, within an individual, when they are performed within a half-hour period?
2. What is the magnitude of the individual variations manifested in the learning functions considered in this investigation?
3. How accurate is the curve that has been selected to describe the norm of learning of each individual?
4. What is the relation between change in level of performance in a number of learning tasks and the magnitude of the deviations?
5. What is the mode of distribution of variations around the individual's norm of learning, and what may be concluded concerning the factors that produce them?

Variability in performance may be produced by the introduction of powerful stimuli, such as excitement, sickness, intoxication. In this investigation we are concerned with the properties of minor deviations, when the conditions of incentive, health, and the like

are as constant as possible, and when the experimental conditions are rigidly controlled. We want to determine whether these minor variations in efficiency are temporary, or whether they persist, influencing performances that follow. Upon the answer to this question hinges our decision concerning the existence of a general factor of efficiency in mental performances, which systematically determines variations in output over a period of time.

# CHAPTER II

## THE LITERATURE

All of the problems in variability to be dealt with in the present study have been attacked in the field of non-learning functions. The literature, however, yields no quantitative information concerning the amount of variability around the norm of learning. According to Hunter (12), "Curves for individual subjects usually fluctuate from practice period to practice period, not because learning is an intrinsically fluctuating process but because the experimenter does not sufficiently control the experimental conditions under which the learning takes place." The quantitative determinations of the degree of fluctuation of the learning score and of correcting for it has, however, not been achieved.

The investigation most germane to our problem is that of H. L. Hollingworth (11). The problem was to determine whether there was any relation between variations in efficiency in the following functions: tapping, three-hole coordination, substitution, steadiness, color-naming and pulse rate. Six male subjects were given 40 practice trials in each function in order to bring them up to a limit. After the approximate elimination of practice effect* the subjects were given 6 trials in each of the functions each forenoon, for seven days. Deviation was measured as the difference between the score obtained and the average performance of the subject. The correlations between fluctuations were found to be very small, though tending slightly to be positive. The median correlation was at .15, the mode at .10–.20, while 32% of the correlations fell below zero. The author concludes that normal deviations are caused by local rather than by general factors.

In order to make our results more comparable to those of Hollingworth, we have extended the analysis of the table of median correlations between changes in efficiency in the functions, as given in the original paper. This table is reproduced here in Table I. We have eliminated the correlations with pulse rate because the process is obviously not related to the other processes, either statistically or functionally. With pulse rate eliminated, the number of

* The gain was approximately 5% of the initial score. It amounted to one-eighth of one per cent per trial. Considering the small reliability of most of the tests and the differences in rate of change of improvement in the different processes, the effect upon resulting correlations should be negligible.

TABLE I

INTRA-INDIVIDUAL INTER-TEST CORRELATIONS. (MEDIANS OF SIX INDIVIDUALS)

(Figure in parenthesis gives median self-correlation of each test)

(*After Hollingworth*)

| | *Tapping* | *Coordi-nation* | *Steadi-ness* | *Substi-tution* | *Color naming* | *Pulse rate* | *Median excluding self-corre-lation* |
|---|---|---|---|---|---|---|---|
| Tapping ... | (.54) | .36 | .20 | .17 | .18 | –.11 | .18 |
| Coordination ........ | .36 | (.62) | .16 | .33 | –.05 | –.02 | .16 |
| Steadiness | .20 | .16 | (.34) | .12 | .15 | –.12 | .15 |
| Substitution ........ | .17 | .33 | .12 | (.33) | .21 | .24 | .21 |
| Color naming ........... | .18 | –.05 | .15 | .21 | (.21) | .12 | .15 |
| Pulse rate | –.11 | –.02 | –.12 | .24 | .12 | (.86) | –.02 |

negative correlations drops to one, from which it follows that a distribution of individual inter-correlations would be predominantly positive, and would show considerably fewer negative correlations than appear in Hollingworth's distribution of correlations, which amounted to 32% of the total amount. In order to find an average value for the various correlations that were calculated, they were transformed into Fisher's (7) z, and averaged. The average z for all of the median correlations in all the functions is .1972. The standard error of z is .0490; $\frac{z}{\sigma_z}$ is 4.025. The corresponding average r for all of the median correlations in all the functions is .1969 ± .0470. These results seem to indicate that the average measure of intercorrelation between the fluctuations is significantly different from zero, and that there is a systematic, though very slight, factor that is common to the variations in the several performances. In addition, it should be noted that the median of a distribution of r's, when the range includes positive and negative values, is likely to be misleading.

Hollingworth's procedure differs from ours mainly in the level of achievement from which variability was measured. In order to exclude the effect of practice, Hollingworth eliminated practice; the influence of practice was excluded in this investigation by the measurement of variability from a curve of learning. This procedure made it possible to obtain measurements of variability *during* learning while the effect of improvement proper was partialled out.

J. C. Flügel (8) attempted to measure the amount of practice, oscillation (this term being synonymous with our use of the term variability) and fatigue in a simple mental function, and to find the relationships of the three measures to each other. The subjects, 46 girls, aged 9 years 5 months to 13 years 5 months, worked addition examples on the Kraepelin addition sheets for 46 successive days, excluding school holidays, each period lasting 20 minutes.

In order to measure the variability at any point on the learning curve, the author subtracted each score from the mean of the chronologically nearest scores, *i.e.*, the scores of the two preceding days, the two succeeding days, and of the day under consideration. It may be observed that this method does not necessarily eliminate the effect of practice from the measure of variability, since there is no reason to suppose that the mean of five points on a rising curve should coincide with the normal value of the middle point. The author appreciated this difficulty, as is evident in the following statement: "The fact that our curves of output were rising rapidly throughout the experiment made it rather difficult to obtain a satisfactory and reliable measure of daily variability." (8, p. 2).

Flügel's study differs from the present one in two respects:

1. No relation between variations in different functions were found because only one function was employed.
2. The effect of learning was most likely not eliminated from the measure of variability.

Thorndike (24) attempted to determine the form of distribution of variability of a number of individuals around their own norms in a number of intellectual tests. This author tested 20 gifted children aged about 10, with fifteen forms of Part 1 of the Thorndike Examination, one each school day. Each score was expressed as a plus or minus deviation from the median score of the individual concerned. In addition, seventy-two children distributed in grades 3–6 were tested at intervals with seven forms of the Thorndike-McCall Test in Paragraph Reading. The forms were approximately equal in difficulty, and no appreciable practice effect was observed. The third part of this investigation consisted of three forms of the same test given in the early fall, mid-winter and late spring, to about 1,400 pupils in the first year of high school. The practice effect was small compared with the differences between trials. The median was calculated for each individual and the other two scores were expressed as deviations from it, while the deviation of the median from itself was not used. The author con-

cludes that "The results . . . are substantially unanimous in fitting the hypothesis that, omitting such extreme conditions (for example, being asleep or being seriously ill) as prevent an individual from being tested at all, depressing conditions are neither more frequent nor greater in their effects than elevating conditions, the real variability being symmetrical."

### *Points of Departure from Previous Investigations*

1. Most studies dealing with variations in mental efficiency have employed non-learning tasks. As far as the writer is aware, there has been no adequate determination of the amount of variation around the curve of learning, and of the mode of distribution of scores of variability in learning.

2. There are no studies reported in the literature, as far as the writer is aware, concerning the relations between variations in efficiency in learning functions within an individual.

3. Most studies in variations of efficiency have employed the method of continuous work. The method adopted in this experiment differs from previous investigations in this respect. Learning trials in the most important functions in this experiment were separated by twenty-four hour intervals. This procedure was decided upon in order to insure that the variability discovered in the course of learning shall be free from the influence of possible episodic factors which may be operative on any single occasion.

# CHAPTER III

## PROCEDURE

### *I. Testing Procedure*

The general procedure of the experiment was as follows :

1. Ten subjects, designated as Group I, were trained daily for 25 successive days, in three tasks of learning. The testing was individual, the subject taking one trial in each of the three tasks daily within a period of a half-hour. The tests were given to the subject at the same time each day. We have records, therefore, of three practice curves for each of our subjects, the scores on each of the corresponding points being obtained within the same half-hour period. At the conclusion of this practice series, the subjects, with the exception of two, returned to practice in a fourth and a fifth learning function, each of which was completed at a *single* sitting.

2. In order to obtain additional information concerning the form of distribution of variations around the norm of learning as well as concerning the relation of variability to ability, a second group of ten subjects, designated as Group II, was engaged several months after this main experiment to take three tests of learning, each learning being completed during a *single* sitting.

### *II. The Tests*

The following tests were selected for our main experiment:

1. A mirror-reading cancellation test.* This test requires the cancellation of all English four-letter words appearing on a sheet between a non-sense arrangement of letters. The material was typed and reproduced by photo-engraving to give an offset impression corresponding to the mirrored image of the original page. The subjects, accordingly, found it necessary to recognize the letters before selecting the proper words in the sheet. The instructions to the subject were: "This is an experiment in reading backwards. Each of the lines below contains a number of four-letter English words. Between the words is a succession of letters of the alphabet making no meaning. You are to find each four-letter word and to

* This is an adaptation of a test of Dr. I. Lorge, of the Institute of Educational Research. It is described in T. C. Publications, No. 438 (1930). The author wishes to thank Dr. Lorge for his helpfulness in the adaptation of the test as well as for other useful suggestions.

enclose it in parentheses. You are to read from right to left. Work as rapidly as possible and try not to omit any words." The time allowed for each practice period was five minutes.

2. A non-sense vocabulary test, consisting of twenty pairs of non-sense syllables. The pairs were selected from the list of Glaze (10); they had 0 per cent association value. They were printed on top of each sheet, while the first of each pair of syllables appeared in the body of the sheet, in random arrangement. The subjects were given the following instructions: "You will find in the upper part of the sheet 20 pairs of syllables. In the body of the sheet you will be presented with a column of the first of each pair of syllables. You are to look at each syllable in the column, find what syllable appears next to it in the key, and write it after the dash following each single syllable. Work as rapidly as you can, without making any errors." The length of each practice period was five minutes.

3. Pyle's symbol-digit test. This test attempts to measure the rapidity with which new associations are formed as a result of repetition. The subject's task is to associate each of 9 simple geometrical figures with a one-place number. The instructions were as follows: "You will find before you at the top of the sheet nine circles. In each circle you will find one of the numbers from 1 to 9, and a symbol, *i.e.,* a simple geometrical figure. In the body of the sheet you will find rows of the same characters, and with empty squares beside them. What you are to do is to write in these empty squares the numbers that correspond with the figures. Work as fast as you can and try to avoid all errors." The length of each daily practice period for Group I was 5 minutes. Group II took 40 one-minute trials with intervals of one-half minute.

4. Cancellation-A test. This test has been previously employed by Cattell, Whitley, Doll, Simpson, Chambers and others. It has 100 A's interspersed among other letters of less frequency. The subjects were given the following instructions: "When I say 'ready,' turn over this sheet of paper, begin at the first line, and mark every A on the page like this." (The subject was shown another marked sample, to make sure that he understood the instructions.) "Mark as rapidly as you can, but try not to leave out any A's. When two A's follow each other in succession mark each separately." This test was given by the time-limit method. The interval between trials was one-half minute, which was previously determined to be sufficient to overcome slight distracting effects.

5. Dot cancellation test. This test, previously investigated by Whitley, consists of 10 lines of dots, 15 dots to each line, equally separated from each other. It has been generally used as a test of speed and accuracy of movement. The instructions to the subjects were as follows: "When I say 'ready' turn over this sheet of paper, and cross out each dot. Work as rapidly as you can, but try particularly to hit the dots precisely." The time allowed was 30 seconds; the interval between trials was 30 seconds.

To summarize more clearly, the following tests were given with the following distributions, to each of the two groups:

TABLE II

LEARNING SCHEDULE OF GROUP I

| *Function* | *Length of Trial* | *Intervals Between Trials* | *Number of Trials* |
|---|---|---|---|
| Non-sense Vocabulary | 5 minutes | 24 hours | 25 |
| Mirror-reading Cancellation | 5 minutes | 24 hours | 25 |
| Pyle's Symbol-Digit | 5 minutes | 24 hours | 25 |
| Dot Cancellation | ½ minute | ½-minute | 40 |
| Cancellation-A | Amount limit | ½ minute | 40 |

TABLE III

LEARNING SCHEDULE OF GROUP II

| *Function* | *Length of Trial* | *Intervals Between Trials* | *Number of Trials* |
|---|---|---|---|
| Cancellation-A | Amount limit | ½ minute | 40 |
| Dot Cancellation | ½ minute | ½ minute | 40 |
| Pyle's Symbol-Digit | 1 minute | ½ minute | 40 |

## *III. Scoring of the Tests*

In scoring Pyle's symbol-digit, the nonsense vocabulary, and the A-cancellation, the number of correct items done was the score. The number of errors was negligible, amounting to less than ½% of the work done. Also, the number of errors decreased slightly with successive trials.

One-half credit was granted for errors in mirror-reading cancellation and in crossing out dots where the error was within one

millimeter. This procedure seemed justifiable in mirror-reading cancellation where subjects would occasionally put in parentheses four letters, three of which were obviously the beginning or end of a four-letter word. When the stroke intended to cross out the dot was one mm. away from it, the subject was granted half credit, since a certain amount of eye-hand coordinating was unmistakably applied, and could not be completely discounted.

It may be added that since most of the comparisons employed deal with the performance of the same subject, that the method of scoring errors enters as a fairly constant factor into successive trials, not affecting relative results. This, coupled with the fact that the number of errors was small and fairly constant from trial to trial, eliminates them as a serious factor in the scoring of our results.

### *IV. Subjects*

There were ten subjects in the main experimental series, which will be designated here as Group I. They were all students of Columbia College, of ages ranging from 19–22, and with the median age at 20. They were all of Jewish extraction, to eliminate possible differences between performances of individuals due to widely varying hereditary and environmental conditions.

The subjects were paid for their services, in order to insure strict compliance with the necessary instructions. They came daily at the same time, including Saturdays, Sundays and holidays, with the exception of two subjects, one of whom was absent on the 8th day, the second on the 20th day. The subjects were not aware of the purpose of the experiment beyond the fact that it was concerned with the measurement of learning ability.

It was indicated to them that maximum application to each task was necessary, that the tests must not be practiced outside of the laboratory, and that they must not absent themselves. The subjects were a highly motivated group, applying themselves eagerly to the work. They returned, with two exceptions, to take the second series of tests (A-cancellation and Dot-cancellation) without remuneration. They made considerable advances throughout the learning, so much so that they kept improving to the very end.

A second group of ten subjects, known as Group II, was given the tests outlined in Table III. Its age range was 17–22, with the average age at 19. All were of Jewish extraction, and students at

the time in the College of the City of New York. It will be observed that each of these tests was completed at a *single* sitting. The results obtained with this group will be utilized for additional information concerning the relation of change of level to variability, the nature of the distribution of variations around the learning curve and the characteristics of the learning curve.

# CHAPTER IV

## METHOD OF CURVE FITTING

### *I. The Problem of Fitting Curves to Learning Data Stated*

Having practiced our group with a number of learning tasks, we obtained 76 curves of improvement in a number of functions. It was our next task to calculate the amount of variability of each individual in each function, during each learning trial, and to proceed with the discussion of the relations between changes in efficiency within an individual.

Variability always implies a norm, from which it may be measured. When dealing with frequency distributions, such functions as the arithmetic mean or median are familiar to us as measures of central tendency from which deviations may be calculated. When, however, we are dealing with a function—like learning—which changes in amount with relation to some other variable, such as time, the mean and median lose their usual significance. Our norm will have to be a rising line or a curve which expresses the tendency of the progress of the individual most closely. The actual scores will then be considered as deviations from this curve of central tendency which serves as a norm. This procedure has been used in economic statistics, where time is an important variable, and lately has been applied to sociological problems, of which the study of Thomas (21) is an illustration.

The trend value at any date is therefore taken to be the *normal* value at that date. It is viewed as the value which would be recorded if the effects of all accidental and complicating processes were excluded, leaving only the effect of learning. This conception of a normal value for any series at a given date, a value which may be used as a base or point of reference in judging the effects of all forces other than the growth factor, is fundamental in the following analysis.

### *II. The Uses of a Curve of Learning*

Aside from its value in the measurement of variability, a curve of learning has a number of important uses in the study of the nature of learning itself. The applications of the learning curve, most of which have been stated by Thurstone (25), will be summarized briefly:

1. Uniformities in the mathematical descriptions of learning data, empirically obtained, might lead to fruitful hypotheses concerning the fundamental relationships obtaining between the variables involved in learning.
2. A correct learning curve would have the effect of correcting the empirical scores for attenuation due to chance errors, as well as to possible systematic non-learning factors, such as variability, which from the point of view of learning may be regarded as errors.
3. It should make possible the prediction of learning performance before it has been attained.
4. It should enable one to measure several distinct phases of learning, such as the rate and amount of previous experience.
5. It should prove useful in determinations of the changes in the form of successive learnings.
6. It may be employed in studies of the modification of the form of learning as conditions of training are varied, a highly important and relatively neglected problem.
7. It is necessary in the analysis of variability during learning, as has been mentioned. It is, therefore, of direct aid in the analysis of the relation between variability in learning and variability in other mental functions.

*III. Methodological Cautions Observed*

The measurement of trend cannot be considered as other than an empirical process. The lack of general principles indicating possible relationships between significant variables in learning, the absence of knowledge concerning the position of the zero point, and in most cases, of the limiting point, not to mention the difficulties involved in the inequalities of the units at different points of the learning, make a decision concerning a convenient curve to describe a series of learning data, difficult and generalizations from it, hazardous. These inadequacies eliminate for the present the possibility of finding the true curve of learning. In order to be able to do that, we would either need more precise knowledge concerning the nature of learning than we have at present, or we would have to assume that we can solve by short-cut mathematical means problems that are essentially psychological.

It appeared best, then, to select a curve that would describe our data as closely as possible, keeping in mind that the selection is to an extent arbitrary, and that the resulting curve will necessarily have the characteristics that were put into it. It also seemed useful, both for theoretical reasons and for the sake of a clearer interpretation of results, to define what assumptions are involved

in the choice of the curve, and not to make any assumptions concerning the true curve of learning, except that it may be similar to the one we have chosen.

In fitting empirical equations extrapolation is invalid unless it may be assumed that the conditions which have affected the series in the past will operate in the future. Consequently, no extrapolation will be attempted in this investigation, nor will the constants of the equation be interpreted with reference to learning characteristics.

*IV. Assumptions Involved in Fitting to Learning Data*

We assume, first, a uniform and consistent trend in learning capable of mathematical expression. We also assume that learning is a continuous function, that learning approaches a limit, and that the rate of change of learning (with the exception of the linear curves) decreases with time and approaches zero. How the rate of change of learning varies with time or with amount of work done, is not known. The problem is to get an expression for it. This expression will of necessity be a guess—a guess, however, whose correctness can be estimated by proper statistical methods.

# CHAPTER V

## THE FITTING OF A CURVE OF LEARNING TO THE PRESENT DATA

### *I. Literature on Curve Fitting Applied to Learning Data*

Among the early attempts to apply an equation to learning data was that of L. L. Thurstone (25). He selected, after many preliminary trials, a hyperbolic form of the second degree, as a most satisfactory representation of learning to typewrite. The equation of the curve is given by $y = \frac{ax}{x + c}$. When learning begins with a positive quantity higher than zero, the equation becomes $y = \frac{a(x + c)}{x + (c + d)}$. This curve is negatively accelerated and approaches a limit *a* as the value of x increases.

Thurstone applied this equation to the learning performances of 83 students at a business school who submitted to one four-minute typing test once a week. The records were taken over a period of a year. Of the 83 students, 32 were eliminated for the following reasons: irregular attendance, unusual irregularity in learning scores, linearity of the curve, delayed positive acceleration and apparent evidence of plateau. The equation was fitted to individual records in the remaining group, with apparently satisfactory results, the learning scores showing considerable correspondence to the score determined.

Blair (2) has pointed out that lack of knowledge of the true zero point in learning will cause the values of the constants of the equation to vary as different portions of the curve are included in the experimental period, or as the amount of previous practice will vary. He criticizes, on this ground, Thurstone's procedure of inferring from the values of the constants to the values of corresponding phases of practice, such as the rate of learning or the limit of learning. Blair's criticism is, in effect, an objection against extrapolation and applies generally to empirical curve fitting. It is unjustifiable to extrapolate to the limit of practice or to the beginning of practice without independent evidence that the conditions determining progress at those extremes are identical with those operative during the experimental period. Does this objection dispense with the usefulness of the empirical curve? It does not do so for two reasons. First, good use may be made of an

empirical fit without interpretation of the constants with reference to learning processes. This investigation is an illustration of the statement. Second, granted that the values of the constants vary as different levels of improvement are included in the practice, the method of curve fitting makes it possible to determine how the values of the constants are affected under certain circumstances and allows the possibility of their correction.

Meyer (16) has proposed as an alternative to Thurstone's hyperbolic curve a trigonometric function of the form

$$t = T + K \text{ arc cot } (n + p)$$

in which T, K, and p are constants, with variables t and n. In the equation, t is the time necessary to perform a unit act, n is the number of the trial, p is the equivalent previous practice, T is the limit which the duration of a test approaches, and K is a constant. To find the values of the constants one guesses at p and K until a satisfactory value for T is found. In view of the fact that the constants have to be guessed at, and in view of Valentine's (26) demonstration that the arc cotangent function approximates the hyperbolic function employed by Thurstone whenever x is beyond some small positive number, slightly above zero, (provided there is no inflection point) it seems preferable to use the hyperbolic form.

Ettlinger (5) has shown that the hyperbola used by Thurstone is a first order approximation to the growth curve $t = \frac{t_z}{1-e^{\frac{-x}{a}}}$, the difference between them being smaller than the errors involved in collecting the data and fitting curves to them. He suggests that the growth curve may be preferable for purposes of stating certain fundamental relationships in learning.

Barlow (1) fitted the hyperbola used by Thurstone to the learning scores of individuals in multiplying two-place numbers by two-place numbers. There were 100 subjects, 14 and 15 years old, who took 16 daily practices for 25 periods. The equation was fitted to the first half of the learning scores of each individual. The constants were fitted by the methods of inspection, of averages, and of least squares. The method of least squares was found to be the most accurate, simple and direct. The author finds that the hyperbolic equations "furnish a reasonably satisfactory fit to the form of the learning curve."

The same equation was also applied by Barlow to the data of Robinson and Heron (20) representing the number of syllables

recalled or anticipated in relation to learning time. The equation represented these scores very satisfactorily.

Jette (13) found that a hyperbolic equation of the form $y = \frac{x}{a + bx}$ best fits the learning scores from 915 Gregg Shorthand Tests. This curve represents the usual negative acceleration. Results of practice with 237 Burnz Shorthand Tests yield a curve which is positively accelerated at the beginning, then passing through an inflection point, becoming negative at the end. Jette fitted to the Burnz Shorthand Tests a curve given by Einstein to represent the variation of the atomic heat of a solid with temperature. This curve furnishes a fair fit to the middle range of the data, while diverging more at both extremes of the learning series.

Chaisson (4) calls attention to certain suggestions by T. B. Robertson (19) regarding the nature of learning and its mathematical expression. Robertson's view on learning were a result of his investigations into the autocatalytic nature of growth. He asserted that the equation $\log_{10} \frac{x}{A-x} = k(t-t')$ well represents growth data when weight is plotted against time. This equation yields an s-shaped curve, in which A is the limit of growth, x is amount of growth at any time, K is a constant, determined by the particular operation, t′ is the value of t when the learning is half over, or when $x = \frac{1}{2}A$. Robertson believed that learning is conditioned by central processes that are autocatalytic in nature.

Chaisson applies this equation to a set of data obtained on himself in learning to type by the sight method, a selection involving 1,668 strokes from a Gaelic translation of the Arabian Nights. There were two practice periods daily, and the number of strokes per minute was the score at each point on the curve. The predicted values were found to be close to the empirical values. Chaisson concludes that it is probable that the true form of the curve is s-shaped, not present in his data because of the high level at which he started.

It may be noted here that similarity between the form of two curves describing the relationship of two independent pairs of events is no valid ground for concluding the processes to be the same for both events. Such a conclusion can be determined only experimentally. Further, the inferences of Chaisson concerning the nature of the progress preceding the experimental training are

uncertain, for the same reason that makes extrapolation from empirical curves generally invalid.

Chaisson also presents a number of curves which Robertson fitted to Ebbinghaus' data on the relation of number of nonsense syllables to number of repetitions necessary to learn them. From the data the positive acceleration imputed to it is not at all certain. Whatever slight evidence of positive acceleration one may find is produced by the first point only. With the first point excluded there remains no trace of a tendency to positive acceleration. Certainly, a decision concerning the form of memorizing should not be based upon such meager evidence.

It may be added that the autocatalytic curve requires a knowledge of the value of the limit of learning. Since most learning experiments are not continued to the limit, the usefulness of this equation becomes considerably curtailed.

## *II. Choice of a Theoretical Equation*

The curve that was chosen to describe our learning data is a hyperbolic form of the second degree suggested by Thurstone (25) and applied by Thurstone and Barlow (25, 1) previously. The equation of the curve is $y = \frac{ax}{x + c}$. When we assume that achievement does not begin at zero, as has been done in this investigation, the equation will read $y = \frac{a(x + c)}{x + (c + d)}$ in which

- $y$ = achievement in terms of the number of successful acts performed per unit time.
- $a$ = limit of practice in terms of attainment units
- $x$ = total number of practice acts since the beginning of training
- $c$ = equivalent previous practice (previous to the beginning of controlled training) in terms of formal practice units.
- $d$ = rate of learning. It is a measure of the rapidity with which the subject approaches his limit.

This equation will start with a positive score at the beginning of practice, for when $x = 0$, $y = \frac{ac}{c + d}$. As practice continues, the value y will approach a limit, a. According to the equation, the rate of change of learning with respect to the variable x is: $\frac{dy}{dx} = \frac{ad}{(x + c + d)^2}$. The numerator of the first derivative is a constant, and the denominator becomes larger and larger as x increases. The derivative, therefore, becomes smaller and smaller, and approaches zero as a

limit. The chosen curve, therefore, agrees with the general features of learning progress in that it is consistent with the presence of a positive amount of knowledge before the controlled learning begins, the presence of a limit in learning and the presence of a decrease in the rate of learning which approaches zero.

Curves which showed a linear trend were fitted by the equation $y = a + bx$, y and x being measured in the same units as the curvilinear trends.

In order to calculate the values of the constants, a, c, and $(c + d)$ the curve was rectified as follows:

The initial achievement value, $Y'$, was expressed in terms of the constants of the equation. This is done by setting $X = 0$. $Y'$, of course, is known, representing the first score of the subject. Now, when $X = 0$, $y = \frac{ac}{c + d} = Y'$. Substituting $Y'$ for the constants and simplifying, the equation becomes $\frac{XY}{Y - Y'} = a \frac{X}{Y - Y'} - (c + d)$. The equation may be restated as $S = m + nT$, where $S = \frac{XY}{Y - Y'}$ and $T = \frac{X}{Y - Y'}$ $n = a$, and $m = -(c + d)$.

This equation is linear when S is plotted against T.

The values of the constants are obtained by the method of least squares. When determined by this method, the values of m and n are as follows:

$$m = \frac{\Sigma(T) \cdot \Sigma(S \cdot T) - \Sigma(T)^2 \cdot \Sigma(S)}{[\Sigma(T)]^2 - n\Sigma(T)^2}$$

$$n = \frac{\Sigma(T) \cdot \Sigma(S) - n\Sigma(S \cdot T)}{[\Sigma(T)]^2 - n\Sigma(T)^2}$$

### *III. Determination of the Constants*

There are a number of different methods available for calculating the constants of an equation, once the equation has been selected. The method chosen will affect the degree to which the theoretical curve will correspond to the obtained scores. The choice of the method will depend upon the degree of accuracy required in the results as well as upon the applicability of the method to the specific material.

The method of inspection was the first to be rejected. Since learning scores possess measurable variability, curves fitted by the method of inspection will be considerably subjective. The results

will differ from one investigator to another and within each investigator from time to time.

The method of moving averages was not adopted, because it is considerably affected by variations at both extremes; the choice of the best period for such an average is more or less arbitrary; and when it is used for correlating two or more series, a correction factor is necessary.

The method of selected points makes use of only a few points in the series, and may consequently distort final results. It was, therefore, eliminated.

We have employed the method of least squares in computing the constants of each curve because it insures that the resulting distribution of theoretical scores will be as close as possible to all of the scores of the obtained data for the curve decided upon. The constants obtained by this method are so determined that the sum of the squares of the deviations of the obtained scores from the curve will be a minimum. It may be demonstrated mathematically that the resulting theoretical values are the most probable for the selected curve. Proofs of the method of least squares are easily available, and will not be repeated here.

### *IV. Choice of Coordinates*

A curve of improvement usually states the relationship of two variables of learning.* It is, therefore, important to select those two phases of improvement which are most significantly related. The coordinates employed here are achievement per unit of time (Y) and amount of work done up to and including each trial (X). Amount of work done has been chosen because it fundamentally determines future achievement. The use of time as the independent variable has validity only because time is taken to be a measure of previous work done. The present selection of coordinates also states both variables in the same units; it will, consequently, allow a much clearer interpretation of correlation results than would have been possible otherwise. That the amount of work done is very significantly related to achievement is shown later.

* A curve of relationship can consist of n variables. However, most learning data are presented so as to reveal the relation between only two variables.

# CHAPTER VI

## RESULTS

### *I. General Results*

Preliminary observation of the results reveals that:

1. There was considerable improvement in all the functions selected.
2. No limit of learning was reached in any of the tests, with the exception of about three individual curves. It should be noted, however, from (1) that the bulk of improvement has occurred within the experimental periods.
3. Of the 76 curves of improvement obtained,
   a. 57 curves showed rapid initial progress, decreasing in rate, and eventually sloping off gradually to a seeming limit.
   b. 19 curves showed a linear trend, *i.e.*, the rate of improvement was practically constant from the first to the last trial.

Figures 1, 2, 3, 4, 5, 6, and 7 represent some of the fits obtained in a number of functions. The continuous curve joins the scores theoretically determined, while the dots represent the corresponding empirical scores.

The number of negatively accelerated and linear curves found in each function are classified in Table IV below.

TABLE IV

NUMBER OF NEGATIVELY ACCELERATED AND LINEAR CURVES

| *Function* | *Symbol Digit Group I* | *Vocabulary* | *Mirror-Reading* | *Dot Cancellation* | *Cancellation A* | *Symbol Digit Group II* |
|---|---|---|---|---|---|---|
| Linear curves | 0 | 2 | 4 | 5 | 7 | 1 |
| Negatively accelerated curves | 10 | 8 | 6 | 13 | 11 | 9 |
| Total | 10 | 10 | 10 | 18 | 18 | 10 |

### *II. Measurement of Variation*

Having determined the value of the constants of the equation of an individual in a function, the next step was to solve for the

theoretical values of Y corresponding to each obtained score. This was done simply by substituting in the general equation the values of the constants and the respective values of X. In this way we obtain two sets of scores for each individual. One, the obtained set of achievement scores; two, a corresponding set of scores determined by the theoretical equation. The difference between an obtained score at a certain point and its corresponding theoretical score is the measure of deviation at that point. On the assumption that the curve chosen is substantially correct, the difference between an individual's obtained score and his theoretical score is a measure of the degree to which he diverged from his own norm of performance. It is, in other words, a measure of the individual's variability at that point.

We shall proceed now to estimate the accuracy of the determination of scores of variation before analyzing the relations between fluctuations within an individual.

### III. *Amount of Relation between Obtained and Theoretical Scores*

The accuracy of each measure of deviation depends directly upon the faithfulness with which the theoretical scores correspond to the obtained scores. It becomes important, therefore, to calculate the relation between the series of obtained scores and the series of theoretical scores for each individual in each function.

The measure that will yield this information will have to indicate what proportion of the individual's variation has been accounted for in the variation of the theoretical scores. If we take the standard deviation as a measure of the amount of variation, the ratio of the sigma of the theoretical scores to the sigma of the obtained scores will be a measure of the *correlation* between the two sets of scores.

That $\frac{\sigma \text{ theoretical scores}}{\sigma \text{ obtained scores}}$ is a measure of correlation can be demonstrated in the following way:

Let y = obtained score

Let y′ = theoretical score

$$d = y - y'$$

$$\sigma_d = \sigma_{(y-y')} = \sqrt{\sigma_y^2 + \sigma_{y'}^2 - 2r_{yy'}\sigma_y\sigma_{y'}}$$

$$\sigma_d^2 = \sigma_y^2 + \sigma_{y'}^2 - 2r_{yy'}\sigma_y\sigma_{y'}$$

$$\text{Or } \frac{\sigma_y^2 + \sigma_{y'}^2 - \sigma_d^2}{2\,\sigma_y\sigma_{y'}} = r_{yy'}$$

Now, $\sigma_d^2 = (1 - r_{yy'}^2)\,\sigma_y^2$

(for a correlation surface that is linear and homoscedastic)

$$\frac{\sigma_y^2 + \sigma_{y'}^2 - (1 - r_{yy'}^2)\sigma_y^2}{2\,\sigma_y\sigma_{y'}} = r_{yy'}$$

$$\sigma_{y'}^2 + r_{yy'}^2\,\sigma_y^2 = 2r_{yy'}\sigma_y\sigma_{y'}$$

$$\sigma_{y'}^2 - 2r_{yy'}\sigma_y\sigma_{y'} + r_{yy'}^2\,\sigma_y^2 = 0$$

$$(\sigma_{y'}^2 - r_{yy'}\,\sigma_y)^2 = 0$$

or $\sigma_{y'} = r_{yy'}\sigma_y$

or $r_{yy'} = \dfrac{\sigma_{y'}}{\sigma_y}$

The correlations are given for each individual in Pyle's symbol-digit, vocabulary learning, and mirror-reading in Table V. The correlations in the other functions are given in Table VI. The results show an exceedingly high relationship between the scores determined by the equation and the obtained scores. We interpret this result to mean that the curve summarizes accurately the progress of learning in the individuals to whom it was applied. The small values of the coefficient of alienation bear testimony to the appropriateness of the curve. It is unfortunate that no quantitative measurements of the correspondence between obtained learning scores and theoretical scores are reported in the literature, with which to compare the present results. Barlow (1) reports in his

TABLE V

INTERCORRELATION BETWEEN OBTAINED SCORES AND SCORES DETERMINED BY THE THEORETICAL EQUATION

| *Subjects* | *Symbol-Digit I* | *Vocabulary* | *Mirror-Reading* |
|---|---|---|---|
| Fr. ........... | .975 | .977 | .982 |
| Zi. ........... | .971 | .988 | .973 |
| Fe. ........... | .981 | .974 | .983 |
| Ro.* ........... | | .990 | .974 |
| Gu. ........... | .980 | .978 | .974 |
| We. ........... | .972 | .986 | .954 |
| Ta. ........... | .934 | .963 | .973 |
| Si. ........... | .977 | .983 | .972 |
| Be. ........... | .973 | .993 | .949 |
| Ab. ........... | .972 | .991 | .979 |

* The curve of improvement of subject Ro. in Pyle's symbol-digit was in general irregular, conforming to no definite shape. It was, therefore, considered best to leave it out of discussion.

study of the form of the practice curve in multiplying two-place numbers by two-place numbers that "while the learning curve forms under consideration fail to show exact agreement with the forms of the learning curve equation, there is sufficient similarity between them to justify the use of the latter for representing the former in certain mental traits."

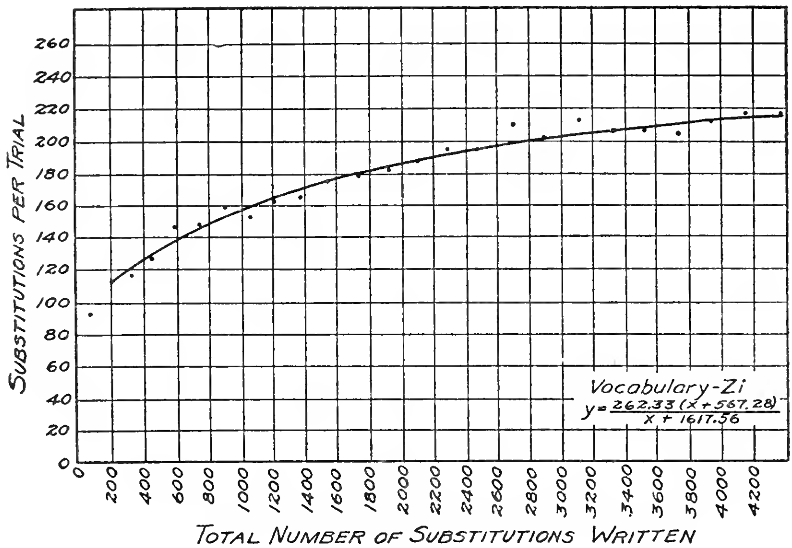
TABLE VI

INTERCORRELATION BETWEEN OBTAINED SCORES AND SCORES DETERMINED BY THE THEORETICAL EQUATION

| *Group II* | *Cancellation—A* | *Dots* | *Symbol-Digit II* |
|---|---|---|---|
| Da. | .866 | .863 | .959 |
| Op. | .834 | .742 | .922 |
| Le. | .802 | .931 | .910 |
| Gr. | .979 | .982 | .930 |
| Wi. | .861 | .846 | .896 |
| Sc. | .943 | .967 | .883 |
| Me. | .875 | .810 | .931 |
| Gra. | .650 | .909 | .910 |
| Mi. | .918 | .662 | .960 |
| Ra. | .799 | .902 | .914 |
| *Group I* | | | |
| Zi. | .929 | .945 | |
| Fe. | .875 | .943 | |
| Ro. | .961 | .843 | |
| Gu. | .706 | .957 | |
| We. | .904 | .789 | |
| Ta. | .650 | .899 | |
| Si. | .958 | .912 | |
| Ab. | .933 | .614 | |

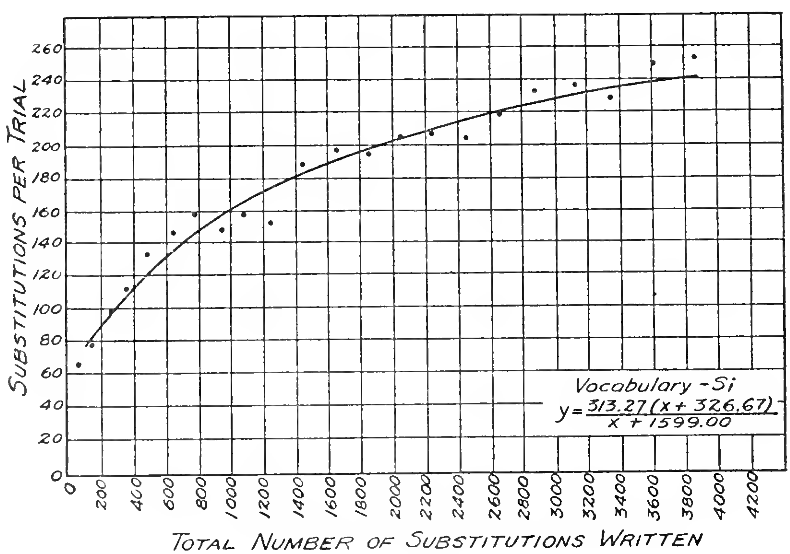
It will be observed that the correlations in Table VI are considerably smaller than the correlations in Table V. The decrease in the degree of correlation would seem to be satisfactorily accounted for by the smaller time-limits within which the tests were administered. All of these learnings were begun and completed in one hour, as contrasted with the five-minute time limits in the first set of tests, which were distributed over 25 days. The decrease of the time spent in learning the second series of tests represented an approximate reduction of $\frac{4}{5}$, which would be expected to decrease the reliability of the results drastically.

The present finding extends the usefulness of the hyperbolic curve to the tasks of symbol-digit substitution, non-sense vocabulary



260
240
220
200
180
160
140
120
100
80
60
40
20
0
SUBSTITUTIONS PER TRIAL
Vocabulary-Zi
$y = \frac{262.33\,(x+567.28)}{x+1617.56}$
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400 3600 3800 4000 4200
TOTAL NUMBER OF SUBSTITUTIONS WRITTEN


Fig. 1



260
240
220
200
180
160
140
120
100
80
60
40
20
0
SUBSTITUTIONS PER TRIAL
Vocabulary-Si
$y = \frac{313.27\,(x+326.67)}{x+1599.00}$
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400 3600 3800 4000 4200
TOTAL NUMBER OF SUBSTITUTIONS WRITTEN


Fig. 2

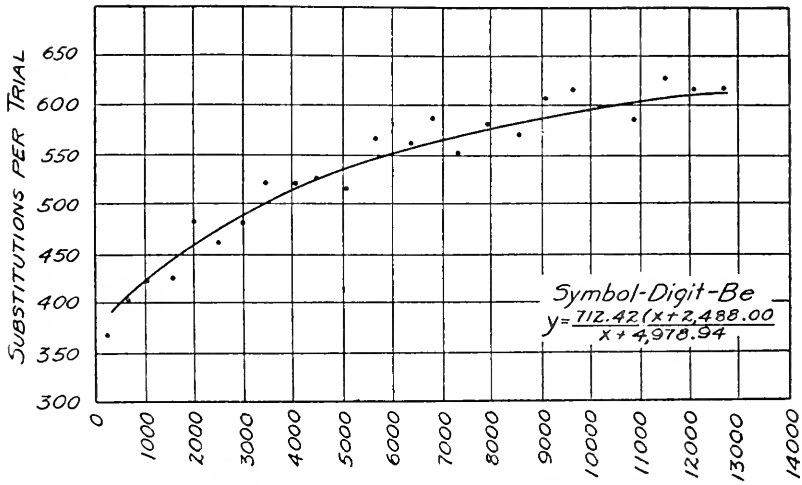



Fig. 3

substitution, mirror-reading cancellation, dot-cancellation, and the A-cancellation test. The applicability of the theoretical equation to these tasks—to most individuals—and to the tasks dealt with by Thurstone and Barlow would suggest as promising procedure a more extensive investigation of the range of tasks which are accurately described by it. It would also suggest a more searching analysis of the causes producing linear curves, and of the possibility of reducing both forms to one common form.

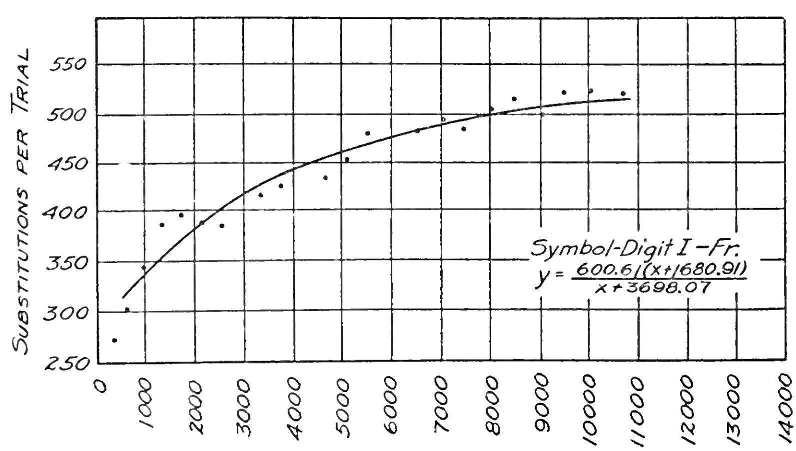
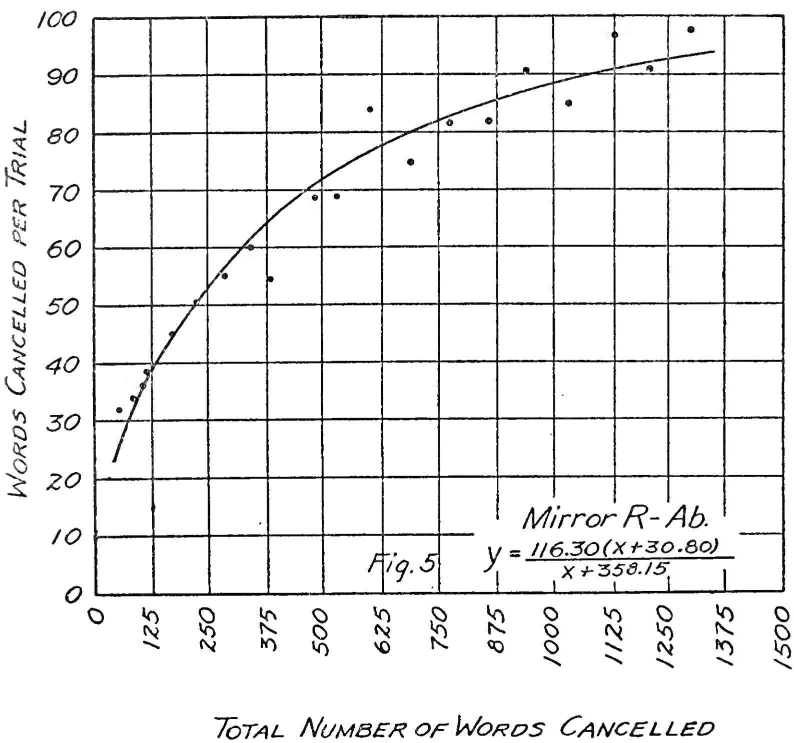
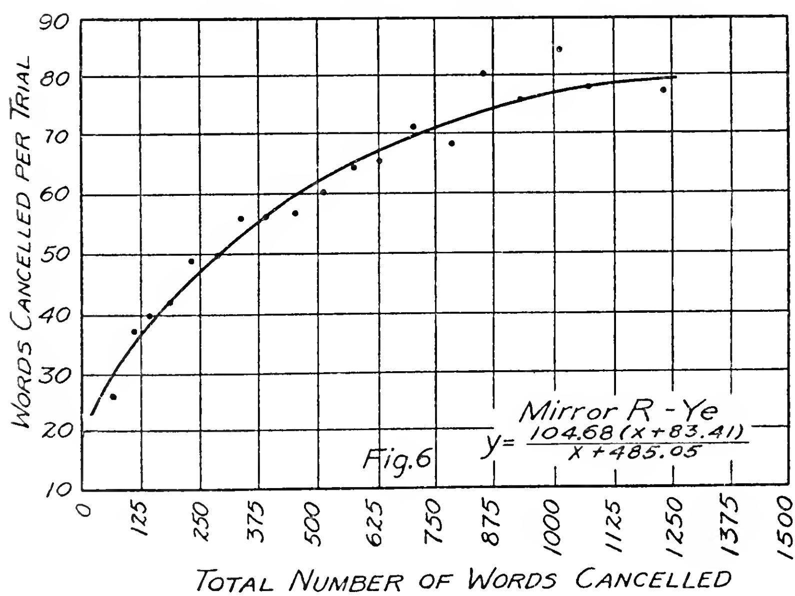



Fig. 4


Words Cancelled per Trial
100
90
80
70
60
50
40
30
20
10
0
0
125
250
375
500
625
750
875
1000
1125
1250
1375
1500
Mirror R-Ab.
$y = \frac{116.30(x+30.80)}{x+358.15}$
Fig. 5
Total Number of Words Cancelled


Words Cancelled per Trial
90
80
70
60
50
40
30
20
10
0
125
250
375
500
625
750
875
1000
1125
1250
1375
1500
Mirror R-Ye
$y = \frac{104.68(x+83.41)}{x+485.05}$
Fig. 6
Total Number of Words Cancelled

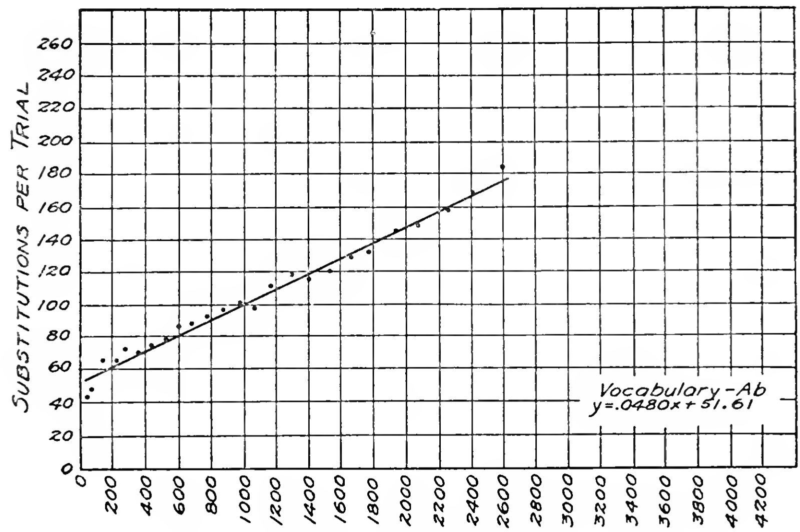


Fig. 7

*IV. Coefficients of Net Determination of the Theoretical Scores by the Obtained Scores*

When the units of X and Y are composed of the same elements of equal variability, all of which are present in X but some of which are lacking in Y, it can be proven that $r^2_{xy}$ measures that proportion of the elements in X which are also present in Y. The measure, $r^2$, is called the coefficient of net determination. Since the X and Y coordinates employed in this equation were amount of work done up to and including the given period against achievement during the given period (which is the reason why they were selected), the coefficients of net determination may be calculated. This measure yields a much more unambiguous statement of the relationship between the two sets of variables than the coefficient of correlation. The coefficient of non-determination or the degree to which Y is not determined by X, is obtained by subtracting the coefficient of determination from unity.

In computing the values of $r^2_{yx}$ we found it more convenient to employ the formula

$$r^2_{yx} = 1 - \frac{\sigma^2 \text{ deviation scores}}{\sigma^2 \text{ obtained scores}}\left(\frac{n-1}{n-m}\right)$$

than the expression $\dfrac{\sigma^2 \text{ theoretical scores}}{\sigma^2 \text{ obtained scores}}$. It can be shown that the

two expressions are equivalent. In the first expression, n = number of cases, m = number of constants, and the term $\frac{n-1}{n-m}$ serves as a correction for the number of observations and the number of constants.

$$\sigma_d^2 = \sigma_y^2 + \sigma_{y'}^2 - 2r_{yy'}\sigma_y\sigma_{y'}$$

$$\frac{\sigma_d^2}{\sigma_y^2} = 1 + \frac{\sigma_{y'}^2}{\sigma_y^2} - 2r_{yy'}\frac{\sigma_{y'}}{\sigma_y}$$

$$\text{But } \frac{\sigma_{y'}^2}{\sigma_y^2} = r_{yy'}^2$$

$$\frac{\sigma_d^2}{\sigma_y^2} - 1 = r^2_{yy'} - 2r^2_{yy'}$$

$$1 - \frac{\sigma_d^2}{\sigma_y^2} = r^2_{yy'}$$

### V. *Standard Error of Estimate of the Theoretical Curves or the $\sigma$ of the Deviation Scores*

Table VII gives the $\sigma_{est.}$, which is also the standard error of the deviations around the curve. It is stated in terms of the units used in measuring the function. The $\sigma_{est.}$ states the amount of discrepancy to be expected in 68% of the cases between the obtained scores and the predicted scores. This measure, however, indicates the size of the residuals without regard to the amount of variation in the dependent variable. The same value of the $\sigma_{est.}$ may be obtained from the testing of two individuals, when one individual may have varied between 100–200, the other between 100–400. The relative importance of the deviations is not given by the $\sigma_{est.}$; that is indicated by the coefficient of correlation between the theoretical and obtained scores, which has been treated previously.

TABLE VII

STANDARD ERROR OF ESTIMATE OF THE THEORETICAL CURVES OR THE $\sigma$ OF THE DEVIATIONS OF THE OBTAINED SCORES FROM THE THEORETICAL SCORES

| | *Symbol-Digit I* | *Vocabulary* | *Mirror-Reading* |
|---|---|---|---|
| Fr. | 12.35 | 10.58 | 5.47 |
| Zi. | 13.62 | 4.63 | 3.61 |
| Fe. | 15.76 | 9.36 | 2.97 |
| Ro. | | 5.45 | 4.88 |
| Gu. | 19.78 | 11.34 | 4.48 |
| We. | 15.84 | 7.68 | 3.77 |
| Ta. | 14.21 | 6.97 | 6.65 |
| Si. | 21.79 | 8.46 | 6.26 |
| Be. | 14.66 | 5.95 | 9.89 |
| Ab. | 19.20 | 4.68 | 4.66 |

In Table VIII are presented the averages of the amounts of the deviations of each individual in each function. Examination of the table will reveal that, when compared with the $\sigma_{av}$, each measure of the average deviation is significantly different from zero.

TABLE VIII

MEAN OF THE DEVIATIONS FROM THE THEORETICAL CURVE. (SIGN NOT TAKEN INTO ACCOUNT)

| | *Symbol-Digit I* | *Vocabulary* | *Mirror-Reading* |
|---|---|---|---|
| Fr. | 10.12 ± 1.54 | 8.95 ± 1.15 | 4.45 ± .71 |
| Zi. | 11.32 ± 1.61 | 3.44 ± .65 | 3.09 ± .50 |
| Fe. | 13.05 ± 1.82 | 8.88 ± 1.12 | 2.37 ± .43 |
| Ro. | | 4.05 ± .78 | 4.08 ± .60 |
| Gu. | 15.02 ± 2.62 | 5.25 ± 2.13 | 3.58 ± .63 |
| We. | 13.27 ± 2.07 | 6.65 ± .80 | 2.85 ± .57 |
| Ta. | 10.58 ± 1.94 | 5.92 ± .75 | 5.98 ± .65 |
| Si. | 19.46 ± 2.04 | 7.13 ± 1.07 | 4.97 ± .85 |
| Be. | 12.75 ± 1.53 | 5.30 ± .63 | 8.48 ± 1.21 |
| Ab. | 14.84 ± 2.53 | 3.90 ± .53 | 3.77 ± 1.23 |

## *VI. Relation Between the Fluctuations of an Individual in Several Functions*

Was there any relation between the deviations of the performance of an individual from his theoretical curve in the several processes that were investigated? Is a high positive deviation in one function more likely to be accompanied by a high positive

TABLE IX

INTERCORRELATIONS (EXPRESSED IN TERMS OF z) BETWEEN FLUCTUATIONS IN EACH PAIR OF FUNCTIONS

| *Subjects* | *Pyle's Symbol-Digit I and Non-Sense Vocabulary Learning* | | *Pyle's Symbol-Digit I and Mirror-Reading Cancellation* | | *Non-Sense Vocabulary Learning and Mirror-Reading Cancellation* | |
|---|---|---|---|---|---|---|
| | z | $\frac{z}{\sigma_z}$ | z | $\frac{z}{\sigma_z}$ | z | $\frac{z}{\sigma_z}$ |
| Fr. | .3273 | 1.4270 | –.2088 | .8853 | –.4825 | 2.1037 |
| Zi. | .2109 | .9427 | .1635 | .6540 | .0569 | .2276 |
| Fe. | .3178 | 1.4905 | –.1404 | .5616 | .1972 | .8361 |
| Ro. | | | .3747 | 1.5438 | .1530 | .5921 |
| Gu. | .3762 | 1.7644 | .5763 | 2.3052 | .1258 | .5032 |
| We. | .2927 | 1.2059 | –.0433 | .1676 | –.2247 | 1.0044 |
| Ta. | .3438 | 1.6124 | .5114 | 2.2297 | .4870 | 2.0649 |
| Si. | .1331 | .5643 | –.1658 | .7030 | –.2865 | 1.1804 |
| Be. | .7762 | 3.4696 | .3648 | 1.6307 | .8888 | 3.7685 |
| Ab. | .3950 | 1.7657 | .4713 | 2.0549 | .0975 | .3900 |

deviation in another function than not? May we speak, in other words, of a systematic factor of efficiency, tending to shift the levels of performance up and down from their norms?

To answer this question, we have paired for each individual the deviations from the theoretical curve for each function with the deviations from the theoretical curve of every other function, by work periods. That is, we paired the deviation in function "a" during a work period "$w_1$" with the deviation in function "b" during the work period "$w_2$," which followed $w_1$. These include correlations between deviations in Pyle's symbol-digit I and non-sense vocabulary learning, Pyle's symbol-digit I and mirror-reading, and non-sense vocabulary learning and mirror-reading. The results are given in Table IX.

### *VII. Interpretation of the Correlation Coefficient Between Fluctuations*

Since, with small samples, the value of the correlation coefficient is often very different from the value in the total population, and the distribution of r is far from normal, the usual tests of the significance of the obtained r are not adequate. We have, therefore, converted each r into Fisher's z, calculated the standard error of z and $\frac{z}{\sigma_z}$. The results are given in Table IX. The number of cases on which each r is based being relatively small, it becomes of interest to know what the *average* correlation for the groups is between fluctuations in each pair of functions. But, since an r is not a linear function of the degree of relationship, an ordinary average is not sufficiently accurate. Furthermore, since the sampling error of the average r is not known, its standard error is also unknown. The average measure of correlation between the deviation in each function for the group was, therefore, obtained by averaging the z's given in Table IX. The significance of each average z was estimated with reference to its standard error.

The results for the average z's are given below in Table X. Our first observation is that the average measure of correlation between the deviations in *each* pair of functions is significant. When we obtain the average of the average z's, the resulting measure of the degree of association between the fluctuations in *all* of the pairs of functions investigated is $.3221 \pm .0273$. This is again a significant result.

TABLE X

Average Intercorrelation Between the Deviations in Pairs of Functions for all Subjects

| *Functions* | *Average z* | $\frac{z}{\sigma_z}$ | *Average r* |
|---|---|---|---|
| Pyle's Symbol-Digit I and Non-Sense Vocabulary Learning | .3559 | 5.0467 | .3411 ± .0414 |
| Pyle's Symbol-Digit I and Mirror-Reading Cancellations | .3093 | 4.3519 | .3002 ± .0430 |
| Non-Sense Vocabulary Learning and Mirror-Reading Cancellation | .3004 | 4.2176 | .2917 ± .0436 |
| Average Correlation between Deviations in all Functions | .3221 | 7.9043 | .3118 ± .0246 |

Inspection of the average measures of relationships between deviations in each pair of functions shows that, while small, they are significant. It may, therefore, be concluded that some correlation exists between the pairs of deviations (as determined by the curve of best fit to all the work periods) in any two functions, when the deviations during two approximately contiguous work-periods are paired. Furthermore, we may infer from the size of the above measures of relationship that the systematic factor of efficiency affects the amount of variability during each work period to an extent of approximately 9–10%. Our analysis of the results of Hollingworth, presented in the review of the literature (p. 8–9), also seems to be in general agreement with the present findings.

### VIII. *Is a Systematic Factor of Efficiency Present?*

Before attempting a final interpretation of these results, it will be necessary to analyze the factors that have tended to produce variability around the theoretical curves of practice.

1. We may first consider errors of a chance nature which are present in any set of data. Under this category we would list sampling errors of measurement, transient changes in physical condition, atmospheric pressure, temperature, etc., as well as errors of measurement due primarily to the unreliability of the test. Chance errors, of course, have the effect of reducing the correlation between two sets of variables.

2. The next possible source of variability are errors produced by the particular curve selected. It has already been stated that

the curve here employed was chosen empirically; it may, therefore, not be the best possible curve for the distribution of scores to which it was applied. Such errors, if they occurred, probably operated as chance factors, since the goodness of fit was approximately equal for the entire range of the learning. That the accuracy of fit remained practically constant is shown in Figures 8, 9, 10, 11, 12, and 13, giving the relation between variability and trials.

3. The last possible source of variability are systematic non-learning factors, such as the presence of a mood, feelings of exhilaration and despondency, which would affect uniformly changes in efficiency over a period of time, and systematic factors of tonus.

If the score of deviation is composed primarily of a multiplicity of events of a chance nature, generally traceable to temporary changes in the condition of the subject, to unreliability of the test, or to errors in fitting the curve of learning, we should expect no correlation between fluctuations in different performances, for individuals or for groups. The finding of correlations between fluctuations which are significant leads to the conclusion that there is present a systematic factor of efficiency within a half-hour period, tending to move the individual uniformly above or below his own norm as determined by the curve of best fit.

The correlations between deviations, while significant, are small. What factors in the situation were likely to affect the size of the correlations? We may first note that the amount of variation in each of the three functions was comparatively small. It consisted, on the average, of *only 2.53%* of the total amount of work done. We have accounted for our results by the selection of highly reliable functions and by the fitness of the selected theoretical curve. Now, the amount of variability in each of the correlated functions being small, and consequently of restricted range, the resulting correlations are smaller than those that would have been obtained if the amounts of variability were larger and of wider range.

### *IX. Relation of Variability to Change in Level of Performance*

Does variability increase or decrease as learning progresses? Will the performance as it improves become more automatic and less variable, as has been often stated, or will the variability remain constant, or decrease?

Thurstone (25), in an investigation of the form of the curve of learning for typewriting, has found that the deviations from the curve of learning increase with practice but that the ratio of the

deviations to the theoretical scores decreases with practice. Our results, as shown in Figures 8, 9, 10, 11, 12 and 13, indicate that the relation of the amount of absolute variability to practice remains approximately constant for the range of time employed in each function, and that the relative deviations decrease with practice. The individual results do not differ from the results for the group. Figures 8, 9, 10, 11, 12 and 13 show the relationship of variability to time for the group. Our results are in agreement with those of Thurstone in finding that the relative deviations decrease with practice; they disagree with the results of Thurstone with regard to the change in absolute deviations with practice. It is difficult to assign a definite reason for the disparity between that portion of Thurstone's results and ours, because of several differences in the conditions of the two investigations, each one of which would be sufficient singly to account for it. The learning function Thurstone dealt with, namely typewriter learning, his distribution of trials as well as the total time range of the experiment, differed widely from ours. The correlations between the theoretical and obtained scores for Thurstone's subjects are not given, so that there is no way of comparing the relative accuracy of the fits. The consistency of our results over an area of five functions as shown in Figures 8, 9, 10, 11, 12 and 13, and similar results obtained by Thorndike (23) in an extensive investigation of the relation of intellectual level to variability, would indicate that they are valid for our learning functions and conditions.

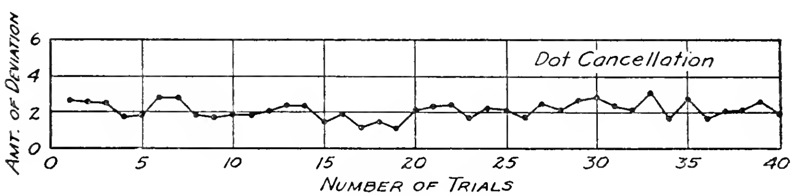


Fig. 8

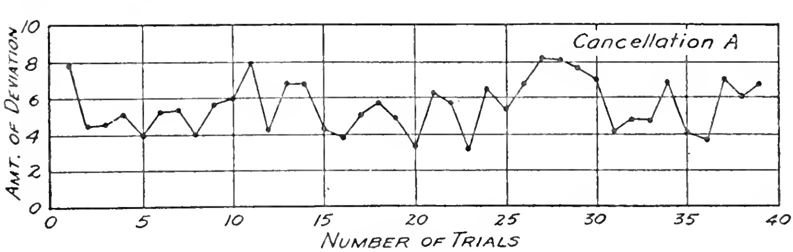


Fig. 9

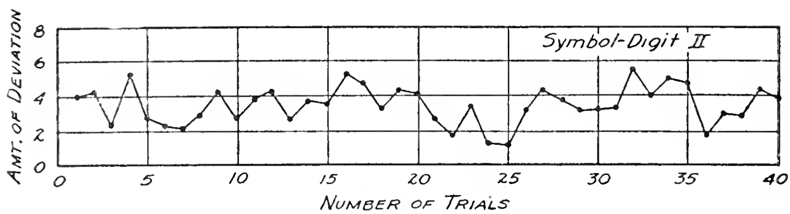



Fig. 10

This finding is of interest in still another connection. It leads to the conclusion that there is no relation between the change in level of performance in each of our learning functions and the amount of absolute variability within the range of the experiment. This result is in harmony with the findings of Thorndike concerning the relation between intellectual level and variability.

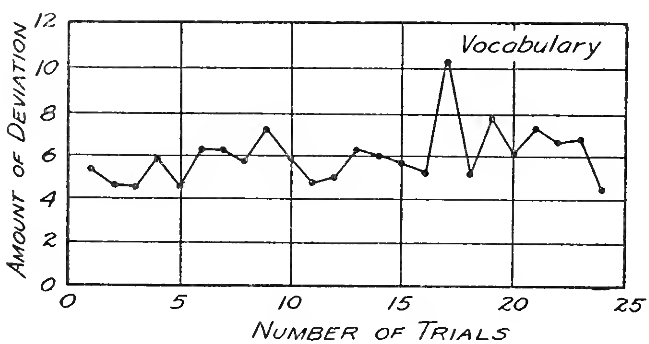



Fig. 11

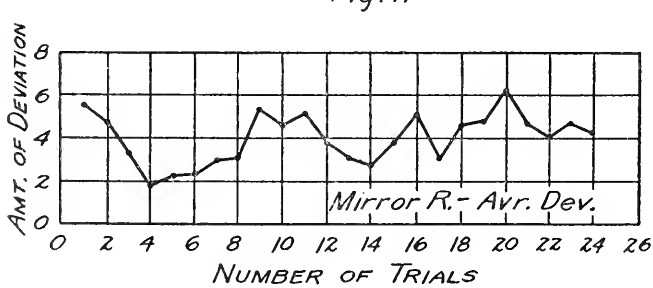



Fig. 12

These results are helpful in confirming still further the accuracy of the correlations between obtained scores and the theoretical scores. The question may fairly be asked, are the correlations between the theoretical and obtained scores the maximum ones

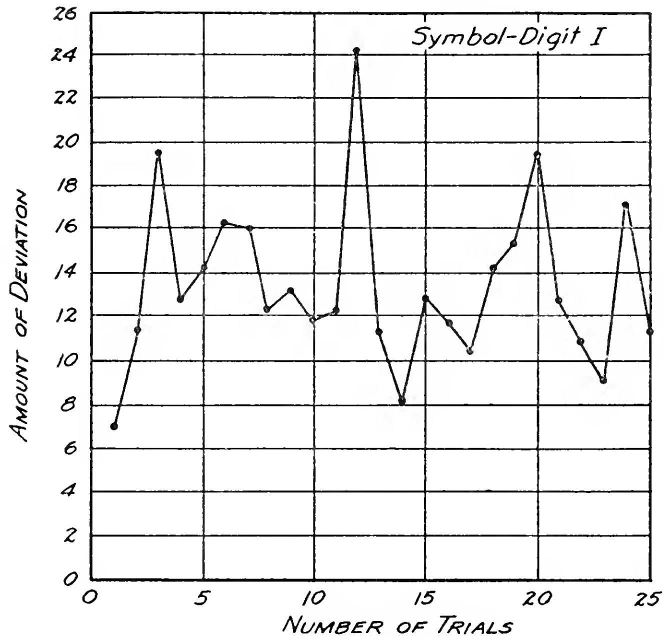



Fig. 13

possible, or would they have been higher with the selection of a more accurate curve? The lack of correlation between the amount of variability and time, or its equivalent here, level of performance, leads to the conclusion that the curves selected have operated so as to eliminate practically all of the learning from the residual variability.

*X. Distribution of the Variations Around the Curve of Learning*

The mode of distribution of the scores of variability around the curve of practice is of considerable importance for the theory of mental measurement. It has a bearing upon the scaling of learning tests in difficulty on the basis of the distribution of records of individuals rather than of groups. The kind of distribution obtained is also suggestive of the factors that may be responsible for the occurrence of variations.

The scores of variability of all subjects in a single function were thrown into one distribution. This was done for each of the functions. Figures 14 and 15 show the distribution of the scores of variation in Pyle's Symbol-Digit II and in Dot Cancellation.

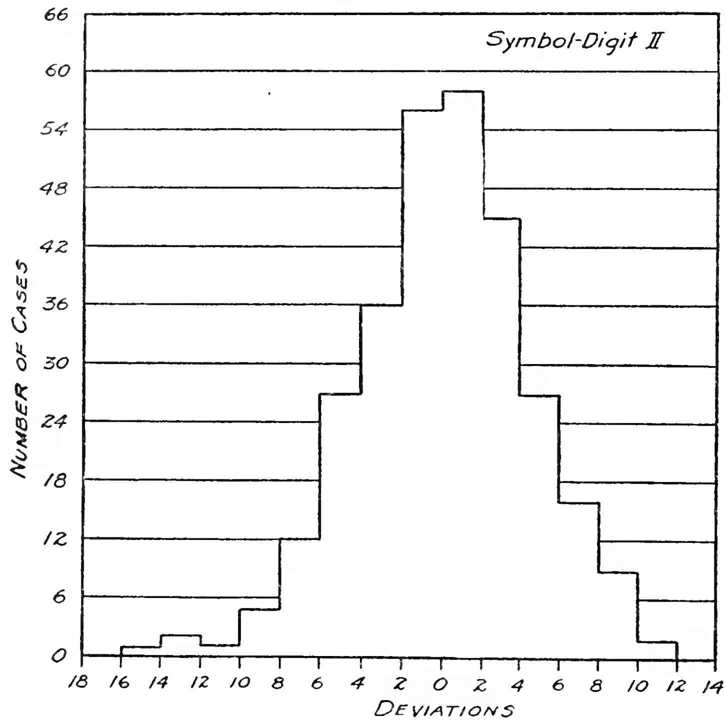



Fig. 14

We have tested for the normality of the distributions by the method of moments. The results are given in Table XI.

TABLE XI

$\beta_1$'s, $\beta_2$'s, and Their Probable Errors

| | $\beta_1$ | $\beta_2$ |
|---|---|---|
| Symbol-Digit I | .0083 ± .0135 | 3.1246 ± .2739 |
| Vocabulary | .0261 ± .3447 | 4.7792* |
| Mirror-Reading | .1278 ± .1003 | 3.3946 ± .5014 |
| Cancellation A | .0488 ± .0923 | 4.2931* |
| Dot Cancellation | .1822 ± .0523 | 3.1884 ± .1895 |
| Symbol-Digit II | .0505 ± .0438 | 3.3416 ± .3457 |

* The values of $\beta_2$ are significantly different from 3.

The scores of variability of the group in each function were transformed into sigma units and the scores of the several distributions were then combined into one distribution. The resulting distribution is plotted in Figure 16.

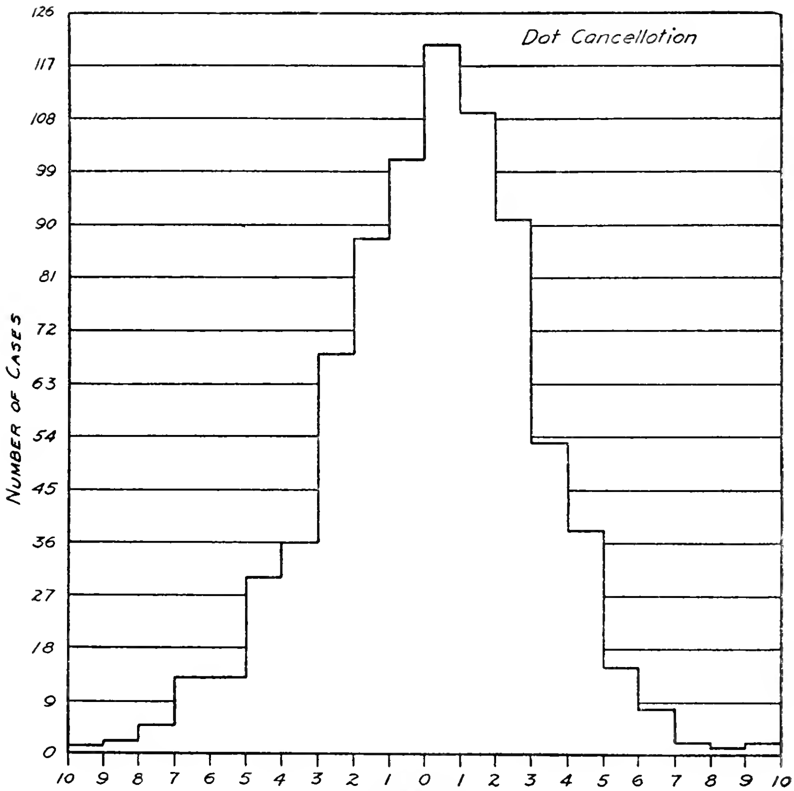



Fig. 15

### *XI. Is There a Typical Curve of Learning?*

What is the bearing of the curve that has been selected to the rate of change of improvement in learning functions, in general? Does the curve hold only for the individuals and the functions to whom it was applied, or has it a more general validity?

The literature reports curves of practice that are negatively accelerated, positively accelerated, and curves that are linear up to a certain point. Most curves of practice show a rapid, continuous rise in the early stages, followed by a more gradual rise, which eventually approaches a limit. The question arises, do the different forms of the curve of practice represent genuine differences in the ways of learning of individuals, or in the types of learning functions, or can they be reduced to a common form with

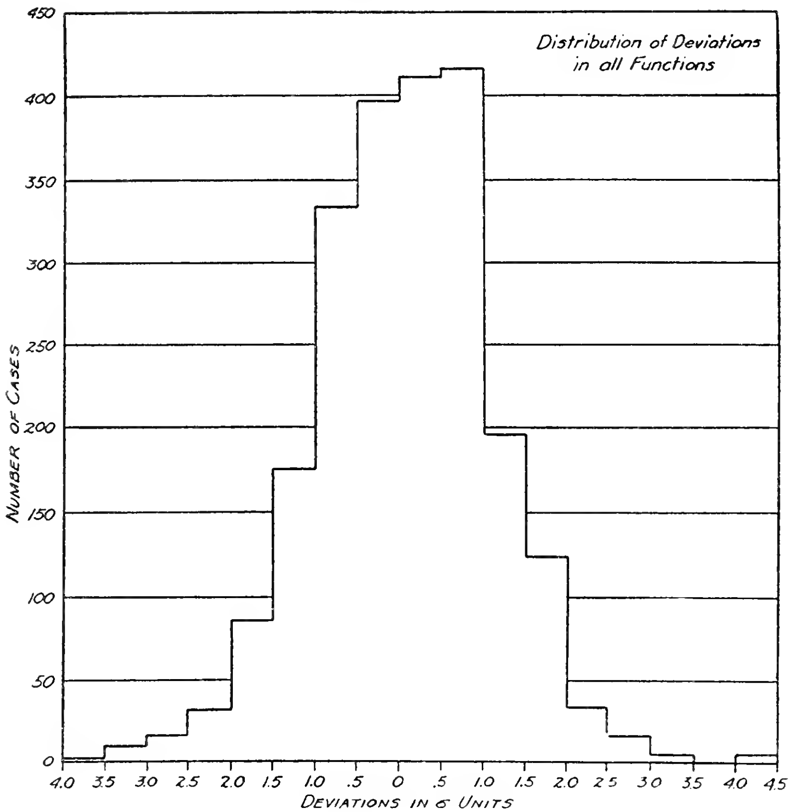



Fig. 16

better control of conditions? And how can the effect of possible disturbing conditions be analyzed?

The preponderance of negatively accelerated curves has led a number of investigators to regard it as the typical or universal form of learning. There are, however, a number of disturbing conditions which undoubtedly enter into most learning situations, affecting the amounts of learning and the form of the curves so that the above statement cannot be accepted at face value. Although 75% of the curves obtained in this investigation are negatively accelerated, the writer is not convinced that they necessarily represent the true form of learning in the functions here treated. It would appear to be of some importance to analyze the operation of these uncontrolled conditions, and to indicate possible methods of dealing with them, before arriving at a final decision.

## *Spurious Factors Affecting Curves of Practice*

### (*a*) *Unequal Difficulty of Individual Items*

Learning curves of any shape or of any combination of shapes may be produced by the proper arrangement of items of a task, which vary in difficulty. The major results of such a situation have been discussed in detail by Thorndike (22). Such curves probably obtain in the learning of school subjects, where the student is likely to encounter from time to time material within the same subject matter, widely varying in difficulty.

The conclusion suggests itself here that the influences exercised by items of varying difficulty on the shape of the curve reflect primarily the eccentricities of the specific subject-matter, which, from the point of view of learning, represent uncontrolled conditions. Only when the learning task consists of items of equal difficulty can the shape of the resulting curve be traced directly to the nature of the learning process, since the factor of difficulty, having been equalized, exerts no relative effect.

Failure to observe this caution seems to be responsible for a certain ambiguity in the statements of some investigators who assert that there is no typical learning curve. It appears as if they were confusing the characteristics of learning with the shifting intricacies intrinsic to tasks of mathematics, physics, and chemistry. We may, therefore, conclude that practice curves are significant only when the elements of the task are of equal difficulty.

### (*b*) *Influence of Past Experience*

The shape of the early part of the curve of practice is likely to be seriously affected by previous experience in the tested function or in related functions, and the greater the amount of previous experience the greater is the influence likely to be. Where there has been previous experience the early scores are to a considerable extent *re-learning* scores. Since evidence from studies of transfer indicate that its effects are likely to operate mostly in the first few trials, the resulting curve of learning will show a very rapid acceleration in the early stages, and, when taken together with later gradual improvement, due to the specific acquisitions in the assigned task, will yield the appearance of a strongly negatively accelerated curve.

A somewhat related attempt to explain the appearance of negative acceleration has been proposed by Book (3). According to

Book, the rapid early rise occurs because the subject is making progress simultaneously along many different lines. When the possibilities for improvement become fewer they also become more difficult, because they are part of a higher hierarchy of habits. The implication is that the easier habits will be first acquired because they are most closely connected with the previous experience of the learner. This view receives some confirmation from Peterson (17) who found that easy items were learned much earlier than more difficult items.

It should be stressed, however, that Book's account does not specify the mode of improvement in each of the components of the task. Book is actually speaking of progress due to *many* learnings. None of these separate learnings need be negatively accelerated in order to produce negative acceleration as a total result. Yet, from the point of view of the fundamental processes involved, it would be more important to determine the nature of the early improvement of these simpler performances than the average result of a complex of performances.

### *A Method of Measuring the Amount of Previous Experience in Learning Functions Suggested*

For the sake of a correct determination of the shape or shapes of learning curves, as well as for the sake of a more precise measurement of learning scores at each trial, it is of the highest importance that the effects of previous experience in each learner be known and taken into account. Obviously, previous experience cannot be eliminated once it has occurred. A tentative experimental method of measuring the amount of previous experience and its precise effect on the shape of the curve and on the magnitude of learning scores will be suggested in the following paragraph.

This method applies to tasks of which several *equivalent* forms can be constructed. To illustrate, construct 25 equivalent forms of a nonsense vocabulary paired associates test. Arrange a learning test consisting of 25 trials, presenting a different equivalent form with each successive trial. If there is any improvement between the first and last trials, it will be the result of past experience and of factors of immediate adaptation to the general character of the material. All specific associations (specific from the point of view of the experiment) have been excluded by the simple device of using equivalent forms. When each learner has reached his limit under the given conditions, administer *one* test for a large

number of trials. The resulting improvement will be due to specific associations acquired. The shape of the curves of practice following the preliminary general practice is more likely to reveal the precise character of learning in that function than a curve obtained on the basis of scores comprising both specific and past learning.

It should be pointed out in this connection that the proposed method can be put to a variety of other uses. In all work involving the relation between initial and final status in learning it is crucial to determine precisely the scores obtained by each individual at those points. As has been indicated in the previous analysis, there are a number of important factors that disturb determinations at the initial points. One is never quite sure whether the first few learning scores are to be referred to the subject's learning ability, to his past experience, or to his initial adaptation to the task. Similarly, whenever one is dealing directly with learning scores, the elimination of the factor of past experience should be found useful.

(*c*) *Inequality of Units at Different Stages of Learning*

If the difficulty values of the units at the successive learning trials do not correspond directly to their present numerical designations, the shape of the curve of practice might be drastically changed if plotted on an equal difficulty scale. It is possible that later achievements, which are at present represented by small increments, might be of considerably greater difficulty than earlier achievements of equal numerical value. If properly represented, such a curve would be likely to show a linear trend showing negative acceleration only near or at the limit. With present methods of scoring, an appearance of negative acceleration may be produced where the real relationships is linear for the bulk of the learning.

(*d*) *Differences in Interest and Motivation*

Where the interest and effort tend to decrease with further practice, a spurious appearance of negative acceleration may be induced. These variables are certainly contributory in certain cases of learning. The findings of Kjerstead, (14) however, that factors of warming up and ennui as measured by comparing two curves obtained one after another, do not change the general form of the curve in syllable learning, would tend to show that in some cases they exert only a minor influence.

### (*e*) *Influence of Type of Coordinates*

It has been pointed out by Thorndike (22) that plotting the same scores of improvement in different frames of coordinates will yield curves differing in shape. It is particularly necessary to keep this in mind when the forms of curves obtained by different experimenters in different functions are plotted. This situation, however, does not imply that curves of practice change their form without rhyme or reason. It serves to emphasize the necessity of standardized procedure in the determination of the shape of the curve.

### (*f*) *Summary*

In conclusion, we may state that the effects of past experience, of differences in the difficulty of units of learning, of progressive differences in interest and effort, are each and all likely to produce disturbances in the shape of the curve. Furthermore, these conditions are likely to produce characteristic negative acceleration where it does not exist. Since these conditions are nearly always present in most learning experiments, and since they are likely to be powerful when they operate, it is suggested that they be taken experimentally into account in order to make possible an answer to the question of the existence of a typical curve of learning. A method has been suggested of eliminating in some cases the effects of past experience. Similarly, it might be profitable to establish a learning scale of equal units in a number of functions. Before these specific steps are taken, it may not be finally asserted that there is or is not a typical curve of learning.

# CHAPTER VII

## DISCUSSION OF RESULTS

It was necessary, in the course of the detailed statement of the methods, results, and interpretations, to deal with each division of the problem in turn, without dwelling sufficiently on the central theme to which they are all related. We are now in a position to bring the several results together, to indicate their relevance to the main problem, and to suggest possibilities of further investigation in the field.

The present investigation is concerned with the measurement of a factor of variability in a number of learning functions. Before attempting to draw any inferences or reach any conclusions from the analysis of the data, it is necessary to judge the raw results on the basis of the adequacy of the measures employed and the control of the experimental conditions. We have selected five tests to be measures of learning in a variety of mental functions. These functions have been traditionally referred to in the psychological literature as eye-hand coordination, perceptual discrimination, associative reaction. What functions these tests actually measure, or how large a sampling of learning ability we have tapped, we are not in a position to say. It will suffice to say that we have measures of learning in five *different* functions.

The tests were given individually, and at the same time each day. The scoring was highly objective, the number of errors being negligible. The subjects formed a highly motivated group. That under the most controlled conditions changes in motivation, yielding fluctuations in output, do occur, goes without saying. These fluctuations, however, are not, from the point of view of our investigation, uncontrolled factors, for they form the object of the study.

Our first concern was the measurement of the variability of each individual in each function. The variability of an individual at any moment, is measured by the degree he has departed from his own norm of performance. A deviation is always a deviation *from* another measurable quantity, which serves as a point of reference; it has no meaning in and of itself. The norm must therefore be known before the deviation from it can be measured.

When the functions selected are, as in the present investigation, learning functions, the norm of performance will be the curve or

line which expresses the true trend of learning, unaffected by all other factors. This trend is nothing other than a mathematical expression of the relation between the amount of improvement and time or amount of previous work done. The psychology of learning has not yet provided us with such general expressions for the description of learning. Nor are there available many principles of learning that might be helpful in determining it. The problem accordingly resolved itself into the empirical selection of a mathematical curve to describe the data precisely.

Of the 76 curves of improvement obtained, 19 were linear. The linearity of these latter curves was established, of course, only for the work periods investigated. If the learning had proceeded to the limit it is likely that the curves would have become non-linear near the end. The linear curves were fitted by a straight line of the form $y = ax + b$. The remaining 57 curves were all negatively accelerated. A hyperbolic curve, previously used by Thurstone and Barlow, (25, 1) was selected and fitted to the negatively accelerated curves.

Nothing is more striking, when surveying the results, than that all of the 76 curves of improvement are adequately described by two simple mathematical expressions. A curve is either negatively accelerated or it is linear. Three-fourths of the curves are negatively accelerated, one-fourth is linear. The extent of individual differences found in *amounts* of improvement finds no parallel when we turn to the *forms* of the curve of practice. This finding, though by no means new, points to the conclusion that the processes responsible for learning in the functions here employed, possess a considerable uniformity.

The question immediately arises: do the linear curves and the negatively accelerated curves represent different ways of learning? If not, which is the true form of the curve? The present results, though suggestive, do not answer either question. We have indicated a number of factors which may be of considerable importance in disturbing both the shape of the curve of practice and the scores of improvement. A method has been suggested of eliminating, in some cases, the effect of previous experience upon learning scores. The question of the existence of a typical curve of learning must, therefore, be left in the present undetermined position. A detailed and thorough study of this problem might be a useful addition to the psychology of learning.

How accurately do the empirical curves of learning describe the trend of improvement? The coefficients of correlation, and particularly the coefficients of net determination between the obtained scores and the predicted scores yield unexpectedly high results. This finding is of significance not only because the hyperbolic curve (and a straight line, in some cases) has been shown to be a reliable expression of learning in five different functions, but also because it is indispensable to all further interpretation of our results. It will be recalled that variation has been stated to be a measure read off from another basic measure, namely, the norm. Had we not arrived at a highly accurate determination of the norms, or of the theoretical learning scores, the derivative measures of variation and their analysis would have led to inconclusive inferences.

Having determined norms of learning and measured the deviations from them, we shall proceed to the problem which is central to this investigation, namely, to the existence of a general factor of variability within our subjects for the given tests. The evidence comes from the correlation between the fluctuation of a learner in different functions. The obtained correlations are significant though small. The average correlation between the fluctuations for all subjects in all functions is $.3118 \pm .0246$. This implies that the factor of efficiency has, on the average, affected the level of variability of each trial to the extent of approximately ten per cent. This result was found after sources of variation were excluded as rigidly as possible through the selection of reliable tests, through the control of the conditions of the experiment, and through the assigning of relatively long learning periods. Had there been more variability in our results, it is likely that the resulting increase of the range of distributions of variability would have led to higher correlations. Our results therefore are more likely to minimize the effect of the factor of efficiency than to exaggerate it. It would seem to be in order to suggest an investigation into the change of the size of the factor of efficiency as the amount of variability is increased.

We have determined the norms of learning, we have measured the deviations from them, and we have found that the deviations between functions which were successively measured yielded significant correlations. The next step was to analyze in greater detail the nature of the obtained deviations. We have first asked, how does the amount of variability change as the individual improves?

The results on this score are clear and consistent. The amount of variation remains approximately the same as the level from which it is measured increases. This is true for each of our subjects in each of the five functions. This result is of particular interest, both because it bears upon the important psychological problem of the relation of variability to ability, and because of the manner in which it articulates with the other results of this investigation.

To consider the latter first. In connection with the correlation between deviations, the following important point of interpretation could have been raised. The obtained deviations have been measured from a curve empirically selected. It is, therefore, likely that the trend of the curve has not included all of the improvement present. The measures of deviation may have included in them factors of practice. Consequently, the correlations between deviations may be affected by a spurious factor of learning present in each of the variables. The present result is the most significant denial of this possibility The finding that there is no relation between amount of variability and change in level is conclusive proof that nothing has been left in the deviation scores that might properly have belonged to the learning.

The importance of this finding goes beyond the implication it has for this investigation. The lack of relationship between variability and level within the individual still needs to be accounted for. It would seem that we have obtained here quantitative confirmation of a much used statement, namely, that an act becomes more automatic with learning.

The only measurable criterion of automaticity is the degree of deviation from a norm. In the present investigation the absolute deviations from the norm of performance remain constant on the average with improvement. It follows that the relative deviations decrease continuously, and that the response becomes more and more automatic with improvement.

It may now be asked whether variations during practice possess the same general characteristics that we find in variations from non-practice performance. The available data allow of such a comparison. The results of Hollingworth (11) and of Thorndike (23, 24) on variability during the absence of practice will be compared with the results of the present investigation. Our results are similar to those of Hollingworth and Thorndike in showing a normal distribution of the variations around their mean. Our results

are similar to those of Thorndike in showing that there is no change in variability with change in the level of performance.* The general characteristics of variation seem to be essentially similar during practice and during the absence of practice, and would suggest that the processes producing variation are the same in both cases. While these comparisons are not entirely conclusive because the two kinds of variability have not been measured in the same group of individuals and in the same functions, the agreement between the results of different investigators, working with different materials and subjects, seems to be most satisfactorily accounted for by the above suggestion.

* The last comparison drawn with Thorndike's results (23) is not a thoroughgoing one. Thorndike found no differences in variability between subjects differing in ability, while we found no differences in amount of variability with *change* in level of the *same* subject. While not strictly analogous, the similarity seems sufficiently striking to be worth mentioning.

## SUMMARY AND CONCLUSIONS

1. The problem was concerned with determining the amount of the deviation of each individual from his own norm of performance in several learning tasks, and with the relation existing between these deviations in the several tasks, with the mode of distribution of such variations, and their bearing upon the validity of a selected theoretical curve of learning.

2. The subjects were 20 college students. Ten were trained in five learning tasks, the other ten in three learning tasks.

3. Training was given at intervals of 24 hours in some functions, and of one-half a minute in other functions. The number of trials was 25 and 40, according to the function.

4. A hyperbolic curve was fitted by the method of least squares to 57 of the 76 obtained curves of practice. A straight line was fitted to the remaining 19 curves. The agreement between obtained and predicted values was found to be very close.

5. The results extend the usefulness of the hyperbolic curve over the five functions dealt with in this investigation.

6. A method is proposed for determining and partialling out the amount of previous experience in a number of learning functions in order to throw light on the existence of a typical curve of learning.

7. The correlations between the deviations of an individual from his own norm in different learning functions, when they are performed within a half-hour period, are found to be significant. It is concluded that there is present a systematic though slight factor of general efficiency, for the subjects and functions dealt with, over half-hour periods.

8. There is no relation between an individual's change of level in a learning function and the amount of variability.

9. The amount of variability remains constant for each individual as learning progresses.

10. Variations during learning have the following characteristics in common with variations during the absence of learning:

(a) the distribution of the scores of variation is symmetrical;

(b) there is no change in the amount of variation as the level of performance changes.

# BIBLIOGRAPHY


1. Barlow, M. C. A Learning Curve Equation as Fitted to Learning Records. Psychol. Rev., 1928, 35, 142–160.
2. Blair, R. V. Thurstone's Method of Study of the Learning Curve. Psychol. Rev., 1918, 25, 81–83.
3. Book, W. F. The Psychology of Skill with Special Reference to its Acquisition in Typewriting. N. Y., Gregg, 1925. 257 pp.
4. Chaisson, A. F. An Alternative Approach to the Mathematical Study of Learning Curves. J. Gen. Psychol., 1930, 4, 352–359.
5. Ettlinger, H. J. A Curve of Growth Designed to Represent the Learning Process. J. Exp. Psychol., 1926, 9, 409–414.
6. Ezekiel, M. Methods of Correlation Analysis. N. Y., Wiley, 1930. 427 pp.
7. Fisher, R. A. Statistical Methods for Research Workers. Oliver, 1925. 239 pp.
8. Flügel, J. C. Practice, Fatigue and Oscillation. Brit. J. Psychol. Monog., 1928, 4, (No. 13) 1–92.
9. Garrett, H. E. Statistics in Psychology and Education. N. Y., Longmans. 1929. 330 pp.
10. Glaze, J. A. The Association Value of Non-Sense Syllables. J. Genet. Psychol., 1928, 35, 255–269.
11. Hollingworth, H. L. Correlations of Achievement Within an Individual. J. Exp. Psychol., 1925, 8, 190–208.
12. Hunter, W. S. Experimental Studies of Learning. *In* Murchison, C. A. Foundations of Experimental Psychology. Worcester, Clark Univ. Press. 1929. 907 pp.
13. Jette, P. L. A Study of the Learning Curves for Two Systems of Shorthand. J. Exp. Psychol., 1928, 11, 145–160.
14. Kjerstead, C. L. The Form of the Learning Curves for Memory. Psychol. Monog., 1919, 26, (No. 116). Pp. 89.
15. Lorge, I. Influence of Regularly Interpolated Time Intervals upon Subsequent Learning. N. Y. T. C., 1930. 57 pp.
16. Meyer, M. F. and Eppright, F. O. The Equation of the Learning Function. Amer. J. Psychol., 1923, 34, 203–222.
17. Peterson, G. M. Negative Acceleration with Material of Varying Difficulty. J. Exp. Psychol., 1928, 11, 40–44.
18. Robertson, T. B. The Chemical Basis of Growth and Senescence. Phil. Lipp., 1923. 389 pp.
19. Robertson, T. B. Principles of Biochemistry for Students of Medicine, Agriculture and Related Sciences. (2nd Revised Edition). N. Y. Lea. 1924. 796 pp.
20. Robinson, E. S., and Heron, W. T. Results of Variations in Length of Memorized Material. J. Exp. Psychol., 1922, 5, 428–441.
21. Thomas, D. S. Social Aspects of the Business Cycle. N. Y. Knopf. 1927. 217 pp.
22. Thorndike, E. L. Educational Psychology. N. Y. T. C., 1913. Volume 2.
23. Thorndike, E. L. Measurement of Intelligence. N. Y. T. C., 1926. 616 pp.
24. Thorndike, E. L. The Variability of an Individual in Repetitions of the Same Task. J. Exp. Psychol., 1931, 6, 161–167.
25. Thurstone, L. L. The Learning Curve Equation. Psychol. Monog., 1919, 26, (No. 114). Pp. 51.
26. Valentine, L. L. A Study of Learning Curves II: The Relationship Between the Hyperbola and the Arc Cotangent Function. J. Gen. Psychol., 1930, 4, 359–362.

[illegible]

150.8
A673
no. 143
1932
Archives of psychology

# ARCHIVES OF PSYCHOLOGY

*List of numbers, continued from inside front cover*

# DIRECTORY OF AMERICAN PSYCHOLOGICAL PERIODICALS

AMERICAN JOURNAL OF PSYCHOLOGY—Ithaca, N. Y.; Cornell University.
Subscription $6.50. 624 pages annually. Edited by M. F. Washburn, K. M. Dallenbach, Madison Bentley, and E. G. Boring. Quarterly. General and experimental psychology. Founded 1887.

JOURNAL OF GENETIC PSYCHOLOGY—Worcester, Mass.; Clark University Press.
Subscription $14.00 per year; $7.00 per volume. 1000 pages annually. (2 volumes.) Edited by Carl Murchison. Quarterly. Child behavior, animal behavior, and comparative psychology. Founded 1891.

PSYCHOLOGICAL REVIEW—Princeton, N. J.; Psychological Review Company.
Subscription $5.50. 540 pages annually. Edited by Howard C. Warren. Bimonthly. General psychology. Founded 1894.

PSYCHOLOGICAL MONOGRAPHS—Princeton, N. J.; Psychological Review Company.
Subscription $6.00 per volume. 500 pages. Edited by Herbert S. Langfeld. Published without fixed dates, each issue one or more researches. Founded 1895.

PSYCHOLOGICAL INDEX—Princeton, N. J.; Psychological Review Company.
Subscription $4.00. 300–400 pages. Edited by Walter S. Hunter and R. R. Willoughby. An annual bibliography of psychological literature. Founded 1895.

PSYCHOLOGICAL BULLETIN—Princeton, N. J.; Psychological Review Company.
Subscription $6.00. 720 pages annually. Edited by Edward S. Robinson. Monthly (10 numbers). Psychological literature. Founded 1904.

ARCHIVES OF PSYCHOLOGY—New York, N. Y.; Columbia University.
Subscription $6.00. 500 pages per volume. Edited by R. S. Woodworth. Without fixed dates, each number a single experimental study. Founded 1906.

JOURNAL OF ABNORMAL AND SOCIAL PSYCHOLOGY—Eno Hall, Princeton, N. J.; American Psychological Association.
Subscription $5.00. 448 pages annually. Edited by Henry T. Moore. Quarterly. Abnormal and social. Founded 1906.

PSYCHOLOGICAL CLINIC—Philadelphia, Pa.; Psychological Clinic Press.
Subscription $3.00. 288 pages. Edited by Lightner Witmer. Without fixed dates (9 numbers). Orthogenics, psychology, hygiene. Founded 1907.

PSYCHOANALYTIC REVIEW—Washington, D. C.; 3617 10th St., N. W.
Subscription $6.00. 500 pages annually. Edited by W. A. White and S. E. Jeliffe. Quarterly. Psychoanalysis. Founded 1913.

JOURNAL OF EXPERIMENTAL PSYCHOLOGY—Princeton, N. J.
Psychological Review Company, 700 pages annually. Experimental. Subscription $7.00. Bi-monthly. Edited by S. W. Fernberger. Founded 1916.

JOURNAL OF APPLIED PSYCHOLOGY—Baltimore, Md.; Williams & Wilkins Company.
Subscription $5.50. 400 pages annually. Edited by James P. Porter. Bi-monthly. Founded 1917.

JOURNAL OF COMPARATIVE PSYCHOLOGY—Baltimore, Md.; Williams & Wilkins Company.
Subscription $5.00 per volume of 450 pages. Three volumes every two years. Edited by Knight Dunlap and Robert M. Yerkes. Founded 1921.

COMPARATIVE PSYCHOLOGY MONOGRAPHS—Baltimore, Md.; The Johns Hopkins Press.
Subscription $5.00. 400 pages per volume. Knight Dunlap, Managing Editor. Published without fixed dates, each number a single research. Founded 1922.

GENETIC PSYCHOLOGY MONOGRAPHS—Worcester, Mass.; Clark University Press.
Subscription $14.00 per year; $7.00 per volume. 1000 pages annually. (2 volumes.) Edited by Carl Murchison. Monthly. Each number one complete research. Child behavior, animal behavior, and comparative psychology. Founded 1925.

PSYCHOLOGICAL ABSTRACTS—Eno Hall, Princeton, N. J.; American Psychological Association.
Subscription $6.00. 700 pages annually. Edited by Walter S. Hunter and R. R. Willoughby. Monthly. Abstracts of psychological literature. Founded 1927.

JOURNAL OF GENERAL PSYCHOLOGY—Worcester, Mass.; Clark University Press.
Subscription $14.00 per year; $7.00 per volume. 1000 pages annually. (2 volumes.) Edited by Carl Murchison. Quarterly. Experimental, theoretical, clinical, and historical psychology. Founded 1927.

JOURNAL OF SOCIAL PSYCHOLOGY—Worcester, Mass.; Clark University Press.
Subscription $7.00. 500 pages annually. Edited by John Dewey and Carl Murchison. Quarterly. Political, racial, and differential psychology. Founded 1929.